School-No: 003089
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Mathematics Analysis and Approaches SL - including German curriculum

(Abitur 2027)

- **The Math SL curriculum is combined with the national curriculum.**
- **The candidates are taught 4 45-minute-classes per week in their first year (Q1) and 5 45-minute-classes per week in their second year (Q2)**
- **Resources:**
  - Coursebook: Mathematics: Analysis and Approaches SL. OUP (2019)
    (chapters mentioned in the course outline refer the coursebook)
  - Lambacher Schweizer Mathematik Qualifikationsphase
    (For topics that are not part of the IB curriculum)
  - IB Mathematics: Analysis and Approaches SL in 80 pages. 2023 Edition
  - Old exam questions: IB Diploma Programme Question Bank
  - Geogebra
  - Graphic Display Calculator CASIO fx-CG 50
- **The IB Math SL-topics that are expanded on the national curriculum topics are marked in green.**
- **The IB Math SL-topics that are not part of the national curriculum at all are marked in blue.**
- **Questions relating theory of knowledge (TOK) and mathematics are marked in purple.**
- **Conceptual links to other subjects are marked in red.**

German curriculum	IB Math SL course curriculum
Q1.1 **Algebra and Calculus: Functions and differentiation (Chapters 2, 3 and 5)** - revision: functional notations, drawing graphs of functions, domain and range of functions (2.1-2.4) - revision: linear and quadratic functions: transformations of functions, graphing quadratic functions, solving quadratic equations with the quadratic formula and the discriminant (3.1-3.7) - the derivative function (5.2) - differentiation rules (power rule) (5.3) - interpretation of first, second and third derivatives (5.4) - Application of differential calculus: optimization and kinematics (5.5) - modelling functions	**TOK: Why do we study mathematics? Can we do without it?** **TOK: To what extent can mathematics be called a universal language?** **TOK: Does a graph without labels have meaning?** - Solving quadratic functions by factorization and completing the square (3.5) **TOK: What is the relationship between real-life problems and mathematical models?** **TOK: How can a mathematical model give us knowledge even if it does not yield accurate predictions?** **TOK: Is mathematics invented or discovered** **Economics: using functions to model and differential / integral calculus to analyze economic problems (eg. production costs, sales development)**

Integral calculus (Chapter 10) - antiderivatives (10.1) - area and definite integrals, properties of definite integrals (10.3) - fundamental theorem of calculus (10.4) - area between two curves (10.5) - applications of definite and indefinite integrals (kinematics and accumulating change)	**Physics: Using kinematics in mathematics and physics** **Geography: modelling problems of sustainable development (eg. Pollution, renewable energy, climate change)** - the indefinite integral (10.1, 10.2) **TOK: How can an infinite area sweep out a finite volume? - Differences between intuition and mathematical knowledge** **Statistics for univariate data (Chapter 6)** - Sampling (6.1) - Presentation of data (6.2) - Measures of central tendency (6.3) - Measures of dispersion (6.4) **Statistics for bivariate data (Chapter 7)** - Scatter diagrams, measuring correlation (7.1-7.2) - Line of best fit (7.3) - Least squares regression (7.4) **Biology / Chemistry: Application of statistical methods in group 4 lab reports** **TOK: Why have mathematics and statistics sometimes been treated as separate subjects?** **TOK: Can you justify using statistics to mislead others? How easy is it to be misled by statistics?** **TOK: How reliable are mathematical measures? (Different measures of central tendency = different properties?)**
Q1.2 **Vector geometry (in fulfilment of the KMK requirement)** - algebraic and geometric approaches - sets of linear equations with more than two unknowns - vector geometry: vector equations of a line (parallel, intersecting, coincident, perpendicular lines) scalar product (angles, magnitude) - vector equations of planes (using parameter and coordinate form) - angles between vectors, lines and planes - measuring distances with normal vectors	- geometry of 3D shapes (11.1) **Sequences and Series (Chapter 1)** - finite and infinite sequences and series, sigma notation, recursive formula (1.1) - limits and convergence (1.1 and 5.1) - arithmetic and geometric sequences and series (1.2-1.3) and applications (1.4) - binomial theorem (1.5) - proofs (1.6) **TOK: Is mathematics a language?** **TOK: What is the difference between reasoning and mathematical proof?** **TOK: Hilbert's Hotel Youtube Clip – What do you think is meant by infinity?** **Physics: Using vectors to model movements**

Q2.1 **Probability (Chapters 8 and 14)** - theoretical and experimental probability (8.1) - tree diagrams (8.4) - conditional probability, dependent and independent events (8.3) - discrete random variables (14.1), probability distributions, expected value, standard deviation - binomial distribution: expectation and variance (14.2)	**TOK: What is the difference between knowledge in statistics and prediction in probability** - Venn diagrams and sample spaces (8.2) **TOK: When can a game be called fair? (Is it fair that casinos should make a profit?)** **TOK: How does belief differ from knowledge?** **TOK: St Petersburg Paradox: how much would you pay to play the game?** **TOK: In what ways may models help or hinder the search for knowledge?** - normal distribution: area under curve, standard normal distribution, inverse normal distribution (14.3) **Economics: Application of probability distributions to economic problems (eg. Consumer behaviour)** **Trigonometry (Chapters 11 and 12)** - right-angles triangle trigonometry (11.2) - the sine rule (11.3) - the cosine rule (11.4) - Applications (11.5) - radian measure, arcs, sectors and segments (12.1) - the unit circle (12.2) - trigonometric identities and equations (12.3) **Physics: bearings, periodic movements**
Q2.2 **Calculus: exponential function (Chapter 9)** - exponents, logarithms (9.1-9.2) - derivatives of exponential functions (chain rule) and the natural logarithmic function (9.3) - modelling exponential functions	**TOK: What are differences between the mathematical and the popular use of the phrase "exponential growth"?** **Biology: population growth** **rational functions (Chapter 4)** - the reciprocal function and its transformation - rational functions (4.1-4.2) - composite functions (2.5) - integrals of composite functions (10.2) - inverse functions (connection to asymptotes, limits, domain and range of rational functions) (2.5) - differentiation rules: chain, product and quotient rule (5.3)

	TOK: Is zero the same as nothing? Why does zero not have a reciprocal? **Trigonometry (Chapters 12-13)** - trigonometric functions (12.4) - derivatives with sine and cosine (13.1) - integration with sine, cosine and substitution (13.3) - kinematics and accumulating change (13.4) - modelling change

Exploration

Year 1: At the beginning of the course the students are introduced to the concept of the mathematical exploration and are asked to watch out for areas of mathematics or concepts they would like to explore.

Year 2: After the Autumn break students are introduced to the details and assessment criteria of the exploration, they use the criteria to assess an example from the support material. Examples of promising and inappropriate topics as well as methods of topic finding are discussed. The students are asked to find the topic and briefly describe the approach they want to take until after the Christmas break. In January / February of Year 2 students are given a period of 3 weeks to work on their first draft. Oral feedback to the first draft is given by the teacher. After that the students are given another 10 days before the final draft is due.

TOK: Explore the relation between real life problems and mathematical concepts and models.

Mathematics and CAS

- Mathematical skills and techniques students learn in the mathematics courses will allow them to evaluate the world around them which will help them to develop, plan and deliver CAS projects. Recent examples in our school where knowledge and skills from in mathematics played a vital role are:
  - planning and carrying out a survey on activities for the next project week
  - Distributing match students and project groups according to personal wishes.
  - systematic planning and calculation of the costs of the graduation ball
  - systematic planning and calculation of the costs of installing and operating a drinking water fountain in the school building.

- CAS projects can also be used by IB students to enrich the mathematics experience of their fellow students or to help students who are struggling with the subject. Recent examples that helped promote other students' involvement in mathematics in our school are:
  - lower school support in Mathematics for students who have recently immigrated

Approaches to learning and development of the learner profile

The approaches to teaching and learning in the Math course focusses on conceptual learning, problem solving and real life applications which among other things helps the students to develop their thinking skills. Open-mindedness and curiosity are needed to think independently and to find new ways to solve problems, while reflection and metacognition help to understand concepts. The methods used often require collaboration in problem solving and the exchange of ideas and approaches. Finally the organization of the course, setting of coursework and the concept of exam preparation help to develop the organizational skills and the affective self-management skills of the students.

The following aspects of the learner profile are explicitly and implicitly developed in the Math course:

- skills to conduct inquiry and to develop in depth knowledge and understanding
- thinking skills to approach complex problems
- communicative skills are developed when concepts are discussed and problems are solved in collaboration with other students
- reflective skills are developed when students consider their own learning experience

Example: exponential functions chapter 9:

Technology to enhance visualization of the graphs of exponential functions

Students are asked to investigate the graphs of exponential functions $f(x) = c \bullet a^{kx}$ in groups by varying the value of a, c and k. Each group focuses on one variable. With the help of the GDC students find out about all the properties of exponential functions, including domain, range, intercepts on the axes, asymptotes, shape and behavior of each graph as x tends to infinity. Students start with definite numbers, make a conjecture, test it and then deduce properties about exponential functions. At the end of the lesson, students present their results to the classmates. In this lesson students are active learners who experiment, question and discover the most important properties of the graphs of exponential functions.